Eur. Phys. J. B **18**, 289–296 (2000)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

The short-time critical behaviour of the Ginzburg-Landau model with long-range interaction*

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Received 9 June 2000 and Received in final form 2 August 2000

Abstract. The renormalisation group approach is applied to the study of the short-time critical behaviour of the d-dimensional Ginzburg-Landau model with long-range interaction of the form $p^\sigma s_p s_{-p}$ in momentum space. Firstly the system is quenched from a high temperature to the critical temperature and then relaxes to equilibrium within the model A dynamics. The asymptotic scaling laws and the initial slip exponents θ' and θ of the order parameter and the response function respectively, are calculated to the second order in $\epsilon = 2\sigma - d$.

PACS. 64.60.Ht Dynamic critical phenomena – 05.70.Ln Nonequilibrium and irreversible thermodynamics

1 Introduction

In recent years, much attention has been paid to the short-time critical dynamics. The short-time phenomena arise at times just after a microscopic time scale $t_{\rm mic}$ needed by the system to remember only the macroscopic condition and to forget all specific microscopic details. The corresponding time regime is also called critical initial slip in order to distinguish it from the uninteresting microscopic time interval between zero and $t_{\rm mic}$. Since the pioneering analytical study of [1], universal short-time scaling has been found in various models (see [2] and [3]). When the system is quenched from a high temperature T_i to the critical temperature $T_{\rm c} \ll T_i$ the order parameter shows in the short-time regime a power law increase $m(t) \sim t^{\theta'}$ with a new universal critical exponent θ'.

The short-time dynamics has been thoroughly investigated for models with short-range interaction (SRI). Since the critical equilibrium properties are modified by the presence of long-range interactions (LRI) it may be interesting to know how the short-time critical behaviour depends upon the interaction range. Experimentally, systems with LRI could be found in ionic solutions where the Coulomb interaction is partially screened [4,5]. The long-range interaction is important in some low-dimensional systems such as the conjugated polymers [6,7].

The statistical mechanics of LRI has a long history. Already in the 60's Jovce [8] studied thermodynamic properties of the static spherical model with long-range ferromagnetic interaction between the spins. The static critical exponents for LRI have been computed for the n-vector model by use of the renormalisation group approach [8–13] and the $1/n$-expansion techniques [14]. There are also Monte Carlo simulations for the one-dimensional static model [15].

The dynamic properties of the LRI in the long-time regime have also been studied as early as the 70's. Suzuki *et al.* [16] extended the dynamic theory developed by Halperin *et al.* [17] to investigate an exponent which describes the critical slowing down in the n-vector model with LRI for $T \geq T_{\rm c}$, with equilibrium initial conditions. Folk and Moser studied a three-dimensional dynamical model for liquids and demonstrated that the critical dynamics was affected by LRI [18]. The kinetic spherical model showed that the short-time critical exponents were modified by LRI [19].

In this paper, we study the short-time critical behaviour of the dynamic Ginzburg-Landau model with long-range exchange interaction. In equilibrium at temperature T the $O(n)$ symmetric Hamiltonian is given by

$$H[s] \equiv \int \mathrm{d}^d x \left\{ \frac{a}{2}(\bigtriangledown s)^2 + \frac{b}{2}(\bigtriangledown^{\frac{\sigma}{2}} s)^2 + \frac{\tau}{2}s^2 + \frac{g}{4!}(s^2)^2 \right\} \tag{1}$$

where $s = (s^\alpha)$ are n-component order parameter fields, τ is proportional to the reduced temperature $T/T_{\rm c} - 1$ and g is the coupling constant. The SRI model corresponds to $a = 1$ and $b = 0$, whereas for the pure LRI model $\sigma < 2$, $a = 0$ and $b = 1$. Since the case $0 < \sigma < d/2$ is covered by a mean-field theoretic description, and since for $\sigma > 2$ and $d > 2$ the model (1) belongs to the same universality

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* Supported by the National Natural Science Foundation of China under the project 19772074 and by the Deutsche Forschungsgemeinschaft under the project Schu 95/9-2.

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class as the SRI model, we will restrict ourselves in the present paper to the range $d/2 < \sigma < \min(2, d)$.

The dynamics to be discussed here, which is called the model A dynamics [20], is controlled by the Langevin equation

$$\partial_t s^\alpha(x,t) = -\lambda \frac{\delta H[s]}{\delta s^\alpha(x,t)} + \xi^\alpha(x,t)$$

where λ is the kinetic coefficient. The random forces $\xi = (\xi^\alpha)$ are assumed to be Gaussian distributed

$$\langle \xi^\alpha(x,t)\rangle = 0;\ \langle \xi^\alpha(x,t)\xi^\beta(x',t')\rangle = 2\lambda\delta^{\alpha\beta}\delta(x-x')\delta(t-t').$$

As mentioned above, the initial state is prepared (macroscopically) at some very high temperature T_i. One assumes that the initial condition $s_0(x) \equiv s(x,0)$ has also a Gaussian distribution $P[s_0] \propto \exp(-H^i[s_0])$ where

$$H^i[s_0] \equiv \int \mathrm{d}^d x \frac{\tau_0}{2}[s_0(x) - m_0(x)]^2 ,$$

τ_0 is proportional to $T_i/T_\mathrm{c} - 1$ and $m_0(x)$ is the (spatially varying) initial order parameter. Being away from criticality ($T_i \gg T_\mathrm{c}$), the initial correlation function will be short-ranged. Since $\tau_0 \sim \mu^\sigma$ (where μ is a renormalisation momentum scale), the physically interesting fixed point is $\tau_0^* = +\infty$, which corresponds to a sharply prepared initial state with initial order m_0 and zero correlation length.

Introducing a (purely imaginary) response field $\widetilde{s}(x,t)$ [21,22], the generating functional for all connected correlation and response functions is given by

$$W[h,\widetilde{h}] = \ln \int \mathcal{D}(i\widetilde{s}, s) \exp\Bigg\{ - \mathcal{L}[\widetilde{s}, s] - H^i[s_0] + \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x (hs + \widetilde{h}\widetilde{s})\Bigg\} \qquad (2)$$

where

$$\mathcal{L}[\widetilde{s}, s] \equiv \int_0^\infty \mathrm{d}t \int \mathrm{d}^d x \times \left\{ \widetilde{s}\left[\dot{s} + \lambda\left(\tau - a\,\nabla^2 + b(-\nabla^2)^{\frac{\sigma}{2}}\right) s + \frac{\lambda g}{6} s s^2\right] - \lambda \widetilde{s}^2 \right\}. \qquad (3)$$

Here we have used a pre-point discretisation with respect to time so that the step function $\Theta(t=0) = 0$. Then the contribution (proportional to $\Theta(0)$) to $\mathcal{L}[\widetilde{s}, s]$ arising from the functional determinant $\det\left[\frac{\delta\xi(x,t)}{\delta s(x,t)}\right]$ vanishes [23].

It is believed that the singularity of the temporal correlation is essential to the short-time scaling and the scaling can emerge in the early stage of the evolution even though all spatial correlations are still short-ranged.

The system is now rapidly quenched to a temperature $T \simeq T_\mathrm{c}$. The order parameter will undergo a relaxation process displaying an initial increase. As long as the correlations are short-ranged and the spatial dimension d is smaller than the critical dimension d_c, the order parameter follows a mean-field ordering process because the mean-field critical temperature $T_\mathrm{c}^{(\mathrm{mf})}$ is larger than the actual critical temperature T_c. This ordering causes an amplification of the initial order parameter. For $d > d_\mathrm{c}$ mean-field theory applies and there is no critical increase.

For the SRI models $d_\mathrm{c} = 4$. The longer is the interaction range, the stronger the suppression of the fluctuations and hence the critical dimension of the LRI model is smaller. Indeed, it turns out that $d_\mathrm{c} = 2\sigma$. Also one would expect that the critical initial increase should be weaker as the interaction range becomes longer.

Since the short-range exchange interaction is irrelevant for $d/2 < \sigma < \sigma_s \equiv 2-\eta_\mathrm{sr}$ where η_sr is the Fisher exponent at the SRI fixed point, one can consider only pure LRI. We apply the ϵ-expansion theory to the LRI model in this regime with $\epsilon \equiv 2\sigma - d$. The critical initial order increase appears in the LRI model for $1 \le d < d_\mathrm{c}$. The scaling behaviour of the critical initial slip is governed by the exponents θ and θ'. They are computed as functions of d and σ.

For σ close to (but larger than) $d/2$ the quantity ϵ is very small and the numerical values of the exponents are accurate when computed to order ϵ^2.

However, when the interaction range is not very long, the situation becomes more complicated, due to a subtle competition between the SRI and the LRI fixed points [10–13] Honkonen [12] computed the β-function of the renormalisation group for the pure LRI model at three-loops and found that the infrared LRI fixed point becomes unstable for $\sigma = \sigma_s$. In the pure LRI model the exchange interaction term is not renormalised, so that the anomalous dimension of the field $s(x,t)$ is zero, whereas in the SRI model the field carries some anomalous dimension γ. Taking the limit $\sigma \to 2$ the expressions for the anomalous dimension (and for other critical exponents) do not coincide. However, as first shown by Sak [10] to the leading non-trivial order and later by Honkonen and Nalimov [11] to all order in $\epsilon' \equiv 4 - d$, the anomalous dimension γ and the other exponents are continuous functions of the parameter σ. This means that the scaling regime of the LRI model is valid only for $\sigma < \sigma_s$, whereas for $\sigma > \sigma_s$ the scaling behaviour is described by the SRI model. At the borderline value $\sigma = \sigma_s$ the two descriptions yield equal values for the critical exponents. Let us conclude here by remarking that these last results were obtained solely for static models.

The paper is organised as follows: in Section 2, the LRI model with $\sigma < \sigma_s$ is studied by the ϵ expansion method. The scaling behaviour of the order parameter, correlation and response functions, as well as the corresponding critical initial slip exponents, are obtained. Section 3 contains conclusions and discussions.

2 The short-time scalings and exponents

Since the SRI is irrelevant for $\sigma < \sigma_s$, in this section we take $a = 0$ and $b = 1$ in (3).

For $g = 0$, the generating functional (2) becomes Gaussian and can be easily evaluated in momentum space. One must take into account the initial condition, by imposing the following boundary conditions:

$$\widetilde{s}(x, \infty) = 0 \qquad\qquad s_0(x) = m_0(x) + {\tau_0}^{-1}\widetilde{s}(x, 0)\ .$$

The free response function $G_p(t,t')$ and the free correlation function $C_p(t,t')$ are respectively

$$G_p(t,t') = \Theta(t-t')\exp[-\lambda(p^\sigma+\tau)(t-t')]$$
$$C_p(t,t') = C_p^{(\mathrm{e})}(t-t') + C_p^{(\mathrm{i})}(t,t'),$$

with equilibrium part $C_p^{(\mathrm{e})}(t-t')$ and initial part $C_p^{(\mathrm{i})}(t,t')$ defined by

$$C_p^{(\mathrm{e})}(t-t') \equiv \frac{1}{\tau+p^\sigma}\exp[-\lambda(p^\sigma+\tau)|t-t'|]$$
$$C_p^{(\mathrm{i})}(t,t') \equiv \left(\tau_0^{-1} - \frac{1}{\tau+p^\sigma}\right)\exp[-\lambda(p^\sigma+\tau)(t+t')].$$

One sets now a perturbation expansion ordered by the number of loops in the Feynman diagrams. It is convenient to consider the Dirichlet boundary conditions $\tau_0 = +\infty$ and $m_0(x) = 0$. The general case is recovered by treating the parameters ${\tau_0}^{-1}$ and $m_0(x)$ as additional perturbations.

The model (2) with Dirichlet boundary conditions must be renormalised. For this purpose notice that the free correlation function simplifies to

$$C_p^{(\mathrm{D})}(t,t') \equiv \frac{1}{\tau+p^\sigma} \times \{\exp[-\lambda(p^\sigma+\tau)|t-t'|] - \exp[-\lambda(p^\sigma+\tau)(t+t')]\}\ .$$

By integrating over the internal momentum and time coordinates one encounters ultraviolet divergences which can be absorbed through the reparameterization of a finite number of coupling constants and fields.

Through dimensional analysis, one can show that the critical dimension $d_c = 2\sigma$ and hence it is convenient to make an expansion in $\epsilon = 2\sigma - d$. We will adopt the dimensional regularisation with minimal subtraction scheme [24] and introduce renormalised quantities through multiplicative factors

$$s_b = Z_s^{1/2} s\ , \qquad \widetilde{s}_b = Z_{\widetilde{s}}^{1/2}\widetilde{s}\ , \qquad \lambda_b = (Z_s/Z_{\widetilde{s}})^{1/2}\lambda\ ,$$
$$\tau_b = Z_s^{-1}Z_\tau\tau\ , \qquad g_b = K_d^{-1}\mu^\epsilon Z_s^{-2}Z_u u\ ,$$
$$\tau_{0b} = (Z_{\widetilde{s}}/Z_s)^{1/2}\tau_0\ , \qquad \widetilde{s}_{0b} = (Z_{\widetilde{s}}Z_0)^{1/2}\widetilde{s}_0 \qquad (4)$$

where the subscript b denotes the bare quantity and $K_d \equiv 2^{1-d}\pi^{-\frac{d}{2}}[\Gamma(d/2)]^{-1}$.

Some comments are in order:

(i) The graphs containing only the equilibrium part of the correlation function are associated to 1PI diagrams and can be made finite by the same renormalisation factors as the translationally invariant theory.

(ii) In addition there are divergences arising for $t+t' = 0$ from the initial part of the correlation function. Remarkably enough, such divergences can be multiplicatively removed if one associates them with n-point connected Green functions. A simple dimensional analysis reveals that new re-normalisations are required only in two-point functions. Due to the Ward identities:

$$s_0(x) = 0 \qquad\qquad \dot{s}_0(x) = 2\lambda\widetilde{s}_0(x)$$

which hold when inserted in the connected Green functions, one is left with a single additional renormalisation constant Z_0.

At a fixed value of σ, a two-loop calculation gives the following renormalisation constants:

$$Z_s = 1\ ; \qquad (5)$$

$$Z_{\widetilde{s}} = 1 - \frac{n+2}{6\epsilon}B_\sigma u^2\ ; \qquad (6)$$

$$Z_u = 1 + \frac{n+8}{6\epsilon}u + \left[\frac{(n+8)^2}{36\epsilon^2} - \frac{5n+22}{36\epsilon}D_\sigma\right]u^2\ ; \qquad (7)$$

$$Z_\tau = 1 + \frac{n+2}{6\epsilon}u + \left[\frac{(n+2)(n+5)}{36\epsilon^2} - \frac{n+2}{24\epsilon}D_\sigma\right]u^2; \qquad (8)$$

$$Z_0 = 1 + \frac{n+2}{6\epsilon}u + \frac{n+2}{12\epsilon^2} \times \left[\frac{n+5}{3} + \left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma\right)\epsilon\right]u^2\ . \qquad (9)$$

Here we have introduced

$$B_\sigma \equiv K_{2\sigma}^{-1}\int\frac{\mathrm{d}^{2\sigma}x}{(2\pi)^{2\sigma}}[1+x^\sigma+(\mathbf{e}+\mathbf{x})^\sigma]^{-\mathbf{2}}\mathbf{x}^{-\sigma}$$

with $\mathbf{e}$ a unit vector in the 2σ-dimensional space, and

$$D_\sigma \equiv \psi(1) - 2\psi(\sigma/2) + \psi(\sigma)$$

with $\psi(x)$ the logarithmic derivative of the gamma function. For the particular case $\sigma = 2$, one has $B_2 = \frac{1}{2}\ln(4/3)$, and $D_2 = 1$. The calculation of the renormalisation constant Z_0 is reported in Appendix A.

According to the general solution of the renormalisation group equation, the renormalised connected Green function of N s-fields, $\widetilde{N}$ $\widetilde{s}$-fields, and M $\widetilde{s}_0$-fields at the fixed point u^* has the following scaling law:

$$G_{N\widetilde{N}}^M(\{x,t\},\tau,\tau_0^{-1},\lambda,u^*,\mu) = l^{(d-\sigma+\eta_s)\frac{N}{2}+(d+\sigma+\eta_{\widetilde{s}})\frac{\widetilde{N}}{2}+(d+\sigma+\eta_{\widetilde{s}}+\eta_0)\frac{M}{2}} \times G_{N\widetilde{N}}^M(\{lx, l^{\sigma+\zeta(u^*)}t\},\tau l^{-\sigma+\kappa(u^*)},\tau_0^{-1}l^{\sigma+\zeta(u^*)},\lambda,u^*,\mu) \qquad (10)$$

where $\eta_s \equiv \gamma(u^*)$, $\eta_{\widetilde{s}} \equiv \widetilde{\gamma}(u^*)$, and $\eta_0 \equiv \gamma_0(u^*)$. The Wilson functions entering the renormalisation group equations are defined by

$$\gamma \equiv \mu\partial_\mu \ln Z_s|_0;\ \beta \equiv \mu\partial_\mu u|_0;$$
$$\kappa \equiv \mu\partial_\mu \ln \tau|_0;\quad \zeta \equiv \mu\partial_\mu \ln \lambda|_0 = \tfrac{1}{2}(\widetilde{\gamma} - \gamma);$$
$$\widetilde{\gamma} \equiv \mu\partial_\mu \ln Z_{\widetilde{s}}|_0;\ \gamma_0 \equiv \mu\partial_\mu \ln Z_0|_0$$

and are computed perturbatively from equations (5–9). The symbol $|_0$ means that μ-derivatives are calculated at fixed bare parameters. For instance, at the two-loop level, the Wilson function γ_0 (related to the initial order parameter) is given by

$$\gamma_0 = -\frac{n+2}{6}\left[1 + \left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma\right)u\right]u\ . \tag{11}$$

By solving algebraically the equation $\beta(u) = 0$ one finds the infrared LRI fixed point

$$u^* = \frac{6\epsilon}{n+8}\left[1 + \frac{2(5n+22)}{(n+8)^2}D_\sigma\epsilon\right] + O(\epsilon^3) \tag{12}$$

and subsequently the values of the Wilson functions at this point.

In order to identify the critical exponents one can compare the standard scaling form of the two-point correlation function

$$G^0_{20}(x-x',t,t',\tau) = |x-x'|^{-(d-2+\eta)} f\left(\frac{|x-x'|}{\xi}, \frac{|x-x'|}{t^{1/z}}, \frac{|x-x'|}{t'^{1/z}}\right)$$

to the equation (10) in which we have set $N = 2$, $\widetilde{N} = M = 0$ and $lx = 1$. Here $\xi \equiv \tau^{-\nu}$.

In this way we find the LRI critical exponents to second order in ϵ [9,16]

$$\eta \equiv 2 - \sigma + \eta_s = 2 - \sigma;$$
$$z \equiv \sigma + \zeta(u^*) = \sigma + \frac{6(n+2)}{(n+8)^2}B_\sigma\epsilon^2;$$
$$1/\nu \equiv \sigma - \kappa(u^*) = \sigma - \frac{n+2}{n+8}\left[1 + \frac{7n+20}{(n+8)^2}D_\sigma\epsilon\right]\epsilon.$$

Notice that the anomalous dimensions of s and $\widetilde{s}$ are $\eta_s = 2 - \sigma + \eta$ and $\eta_{\widetilde{s}} = \eta + 2(z-\sigma)$ respectively, whereas that of the initial order parameter η_0 is given by equation (11) at the fixed point (12).

Employing a short-time expansion of the fields $s(x,t)$ and $\widetilde{s}(x,t)$, as done in [1], one can derive the following behaviour of the full response and correlation functions for $t > 0$ but $t' \to 0$:

$$G(p,t,t') = p^{-2+\eta+z}\left(\frac{t}{t'}\right)^{\theta} f'_G(p\xi, p^z t)$$
$$C(p,t,t') = p^{-2+\eta}\left(\frac{t}{t'}\right)^{\theta-1} f_C(p\xi, p^z t)\ . \tag{13}$$

Here we defined the initial slip exponent θ and computed it to second order in ϵ

$$\theta \equiv -\frac{\eta_0}{2z} = \frac{\epsilon(n+2)}{2\sigma(n+8)}\left\{1 + \left[\frac{7n+20}{(n+8)^2}D_\sigma + \frac{12\ln 2}{\sigma(n+8)}\right]\epsilon\right\}.$$

Let us discuss now the scaling form of the order parameter which relaxes from a non-zero initial value m_0 to zero. As mentioned above we can consider $m_0(x)$ an additional time independent source coupled to the initial response field $\widetilde{s}_0(x)$. Owing to the renormalisation of the initial order parameter

$$m_{0b}(x) = (Z_0 Z_{\widetilde{s}})^{-1/2} m_0(x),$$

no new renormalisation is required for the time-dependent order parameter $m(x,t) \equiv \langle s(x,t)\rangle|_{\widetilde{h}=h=0}$. By taking a homogeneous source $m_0(x) = m_0$, but keeping still $\tau_0^* = +\infty$, we obtain the power law

$$m(t) = m_0 t^{\theta'} f_m\left(m_0 t^{\theta' + \frac{d-2+\eta}{2z}}, \tau t^{\frac{1}{\nu z}}\right) \tag{14}$$

where the exponent θ' is defined by

$$\theta' \equiv -\frac{\eta_s + \eta_{\tilde{s}} + \eta_0}{2z}\ .$$

To second order in ϵ it has the value

$$\theta' = \frac{\epsilon(n+2)}{2\sigma(n+8)}\left\{1 + \left[\frac{7n+20}{(n+8)^2}D_\sigma + \frac{12\,(\ln 2 - \sigma B_\sigma)}{\sigma(n+8)}\right]\epsilon\right\}.$$

As first indicated in [1] for the SRI model, the critical exponents θ and θ' are related by $\theta' = \theta + (2 - z - \eta)/z$.

The function $f_m(x,y)$ appearing in (14) has a universal behaviour at the critical point $\tau = 0$: $f_m(0,0)$ is finite; while for $x \to \infty$, $f_m(x,0)$ behaves like $\sim 1/x$.

3 Discussions and conclusions

When the long range interactions are dominant, $\epsilon = 2\sigma - d$ is small enough and the calculated values of θ' and θ for physical dimensions are numerically reliable. For instance, we list in Table 1 the values corresponding to $\epsilon = 0.1$ for $n = 1$ and $d = 1,\ 2,\ 3$.

Table 1. The values of θ', θ to $\epsilon = 0.1$ for $n = 1$ and $d = 1,\ 2,\ 3$.

	$d=1,\ \sigma=0.55$	$d=2,\ \sigma=1.05$	$d=3,\ \sigma=1.55$
θ'	0.0383	0.0180	0.0117
θ	0.0408	0.0187	0.0120

In the following, we will focus the discussion on θ'. Let us first notice that both the response and the correlation functions measure the fluctuations of the order parameter fields. Since θ and θ' are positive, one expects, according

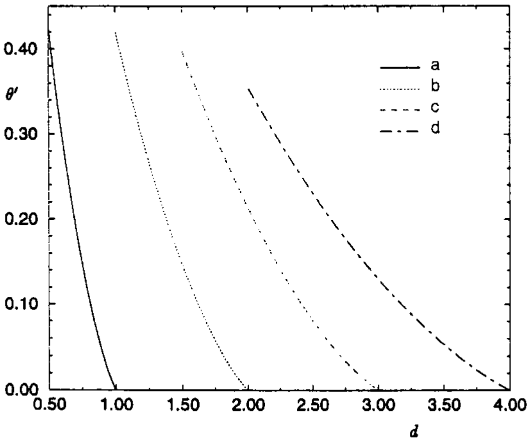

Fig. 1. The exponent θ' for $n = 1$ is plotted *versus* d. Curves 'a', 'b', 'c', and 'd' correspond to $\sigma = 1/2,\ 1,\ 3/2$ and 2 respectively.

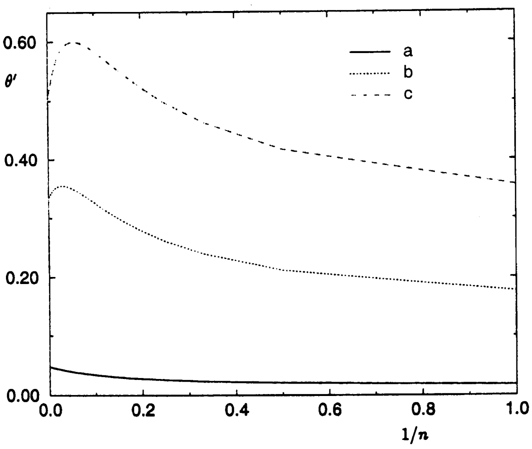

Fig. 2. The exponent θ' for $d = 2$ is plotted *versus* $1/n$. The curves 'a', 'b', and 'c' correspond to $\sigma = 1.05$, 3/2, and 2 respectively.

to equations (13, 14), an initial increase of the fluctuations. Of course, the increase depends upon σ and d.

In Figure 1 the exponent θ' is plotted *versus* d for $\sigma = 1/2,\ 1,\ 3/2$ and 2 respectively and $n = 1$. The value $\sigma = 2$ corresponds to the SRI model. At fixed σ, the exponent θ' decreases when d increases, because fluctuations are reduced as the dimension gets larger. At the critical dimension $d_\mathrm{c} = 2\sigma$ the value of θ' becomes equal to zero. Here other scaling laws would replace the power law.

Figure 2 shows that θ' for $d = 2$ and small σ monotonously increases with n. For larger σ it first reaches a peak and then it decreases toward $n \to \infty$. For other values of the spatial dimension d the pictures are similar. The increase of θ' can be easily understood as more internal degrees of freedom (larger n) help the fluctuations increase. Hence the critical behaviour is smooth in n and can be studied in an $1/n$-expansion. But for large σ one can reach the opposite effect [25], the fluctuations decrease when n exceeds some threshold value. This could be explained by assuming that the interactions among a huge

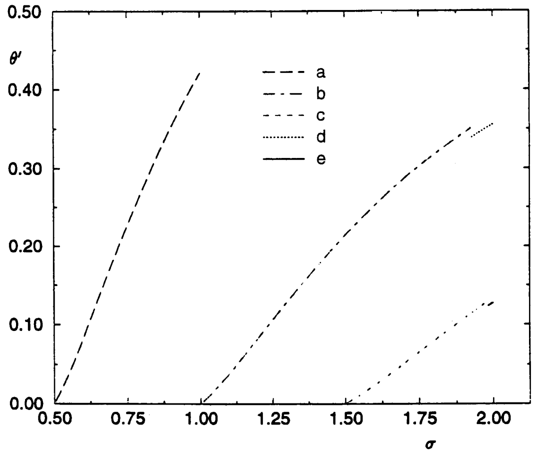

Fig. 3. The exponent θ' for $n = 1$ is plotted *versus* σ. The curve 'a' corresponds to $d = 1$. The curves 'b' and 'c' are drawn for $\sigma \le \sigma_s$ at $d = 2$ and $d = 3$, respectively; the curves 'd' and 'e' are based on equation (18) and describe the behaviour in the region $\sigma \ge \sigma_s$.

number of internal degrees strengthen the mean-fields, but suppress the fluctuations.

In Figure 3 the exponent $\theta^{'}$ is plotted *versus* σ for $n = 1$ and $d = 1, 2, 3$. At fixed d the exponent θ' decreases when σ decreases, because the fluctuations are more suppressed by interactions of longer range (σ smaller). In one dimension, there is no SRI fixed point hence only the curve controlled by the LRI fixed point is observed.

All the previous considerations were of qualitative nature since they do not take into account the change of the fixed point *via* σ. Of special interest in this respect is the LRI fixed point (12). At $\sigma = \sigma_s \equiv 2 - \eta_\mathrm{sr}$ (where $\eta_\mathrm{sr} \equiv \dfrac{n+2}{2(n+8)^2}\epsilon'^2$ and $\epsilon' \equiv 4 - d$) and fixed d we have

$$\begin{aligned}
\epsilon &= \epsilon' - \frac{n+2}{4(n+8)^2}\epsilon'^2;\\
u^* &= \frac{6\epsilon'}{n+8}\left[1 + \frac{3(3n+14)\epsilon'}{(n+8)^2}\right] \equiv u^*_\mathrm{SR};\\
\theta' &= \frac{\epsilon'(n+2)}{4(n+8)}\left[1 + \frac{6\epsilon'}{n+8}\left(\frac{n+3}{n+8} + \ln\frac{3}{2}\right)\right] \equiv \theta'_\mathrm{SR}.
\end{aligned} \tag{15}$$

Here the subscript SR means short-range regime.

In order to explore the limitation of $\sigma \to 2$ of the LRI, we make a double expansion in ϵ and $\alpha \equiv 1 - \sigma/2$ with α of the order ϵ or smaller. The infrared fixed point to order ϵ^2 is located at

$$u^*_\mathrm{wlr} = \frac{6\epsilon}{n+8}\left\{1 + \frac{\epsilon}{(n+8)^2}\left[3(3n+14) + (n+2)\frac{\alpha}{\alpha+\epsilon}\right]\right\}. \tag{16}$$

Here the subscript wlr (weakly-long-range) means that α is at most of order ϵ. The critical initial slip exponent in the weakly-long-range limit can be also computed to this order:

$$\theta'_{\rm wlr} = \frac{\epsilon(n+2)}{4(n+8)} \times \left\{1 + \alpha + \frac{\epsilon}{n+8}\left[6\left(\frac{n+3}{n+8} + \ln\frac{3}{2}\right) + \frac{n+2}{n+8}\frac{\alpha}{\alpha+\epsilon}\right]\right\}. \quad (17)$$

Since α is actually of order ϵ^2 it can be set to zero in (16) and (17), and taking into account that $\epsilon = \epsilon' + O(\epsilon^2)$, one gets

$$u^*_{\rm wlr} = \frac{6}{n+8}\left[\epsilon + \frac{3(3n+14)\epsilon'^2}{(n+8)^2}\right];$$
$$\theta'_{\rm wlr} = \frac{n+2}{4(n+8)}\left[\epsilon + \frac{6\epsilon'^2}{n+8}\left(\frac{n+3}{n+8} + \ln\frac{3}{2}\right)\right]. \quad (18)$$

Clearly the difference between the weakly-long-range and short-range regimes comes from the difference between ϵ and ϵ' as given by equation (15). From the work of [12] we know that the LRI fixed point becomes instable at $\sigma = \sigma_s$. The signal of instability appears however at three-loops. Our work shows that already at two-loops a new fixed point develops, driving the pure LRI model to the intermediate weak-long-range regime. The σ dependence of the critical exponent θ' in this regime is linear and is shown in the curves d and e of Figure 3.

We summarise now our results. We studied the short-time critical behaviour of the Ginzburg-Landau models with LRI in the ϵ-expansion up to two-loop order. We observed an initial critical increase for dimensions smaller than $d_{\rm c}$ and for the interaction range $d/2 < \sigma < d$. We obtained the universal critical exponents θ and θ' of the initial slip as functions of d, n, and the interaction range parameter σ. The limit in which pure LRI is approaching the SRI has been also discussed in some detail.

Finally, our results may be checked by Monte Carlo simulations (*e.g.* the method of [15]) as in the case of the SRI [2]. For experimental purposes, the temperature quenched to $T_{\rm c}$ can be stabilized by a magnetic field firstly applied, and then removed. The experiments which try to test our results may be carried out in physical systems such as ionic solutions and the conjugated polymers.

The authors are grateful to H.J. Luo and B. Zheng for fruitful discussions, and thank C. Untch for help in using the computers.

Appendix A: The calculation of Z_0

In order to determine the renormalisation constant Z_0, we calculate the two-point function $\langle s(-q,t)\widetilde{s}(q,t')\rangle$, with one leg attached to the initial surface $t' = 0$

$$\langle s(-q,t)\widetilde{s}(q,0)\rangle = \int_0^\infty {\rm d}t' \langle s(-q,t)\widetilde{s}(q,t')\rangle^{(\rm e)} \Gamma^{(\rm i)}_{10}(q,t'),$$

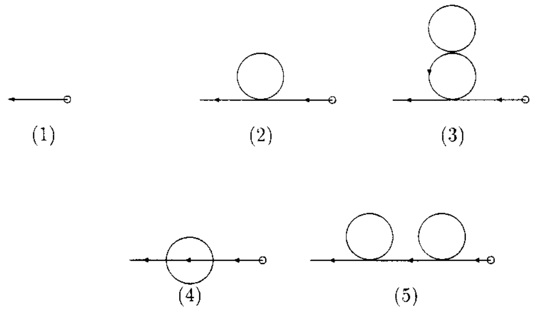

Fig. 4. Diagrams contributing to $\Gamma^{(i)}_{10}(q,t)$ up to two loops.

by using the graphs of Figure 4.

In these diagrams $C^{(\rm D)}_p(t,t')$ and $G_p(t,t')$ are represented by solid lines with and without arrows respectively. The small circle means that one time argument is set equal to zero. The factor $\langle s(-q,t)\widetilde{s}(q,t')\rangle^{(e)}$ denotes the contribution to the two-point function coming only from the equilibrium part $C^{(\rm e)}_p(t,t')$, whereas the residual factor $\Gamma^{(\rm i)}_{10}(q,t')$ is the sum of the amplitudes with at least one initial part $C^{(\rm i)}_p(t,t')$.

We write the singular part of $\Gamma^{(\rm i)}_{10}$ at the critical point $\tau = 0$ in the form

$$\Gamma^{(\rm i)}_{10}(q=0,t) = I_1 + I_2 + I_3 + I_4 + I_5 \quad ({\rm A.1})$$

where I_j with $j = 1,2,3,4,5$ is the contribution of the jth diagram in Figure 4. These contributions are given by

$$I_1 = \delta(t);$$
$$I_2 = -\lambda g\frac{n+2}{6}\int\frac{{\rm d}^d p}{(2\pi)^d}C^{(\rm i)}_p(t,t);$$
$$I_3 = 2(\lambda g)^2\left(\frac{n+2}{6}\right)^2\int_0^t {\rm d}t'\int\frac{{\rm d}^d p}{(2\pi)^d}C^{(\rm i)}_p(t,t) \times\int\frac{{\rm d}^d p'}{(2\pi)^d}G_{p'}(t,t')C^{(\rm D)}_{p'}(t,t');$$
$$I_4 = (\lambda g)^2\frac{n+2}{6}\int_0^t {\rm d}t'\int\frac{{\rm d}^d p}{(2\pi)^d}\frac{{\rm d}^d p'}{(2\pi)^d} \times G_{p+p'}(t,t')\left(2C^{(\rm i)}_p(t,t')C^{(\rm e)}_{p'}(t,t') + C^{(\rm i)}_p(t,t')C^{(\rm i)}_{p'}(t,t')\right); \quad ({\rm A.2})$$
$$I_5 = (\lambda g)^2\left(\frac{n+2}{6}\right)^2\int_0^t {\rm d}t' \times\int\frac{{\rm d}^d p}{(2\pi)^d}\frac{{\rm d}^d p'}{(2\pi)^d}C^{(\rm i)}_p(t,t)C^{(\rm i)}_{p'}(t',t'). \quad ({\rm A.3})$$

By using the formulae

$$\int_0^\infty {\rm d}x x^{\nu-1}{\rm e}^{-\mu x} = \mu^{-\nu}\Gamma(\nu);$$
$$\int_0^t {\rm d}x x^{\nu-1}(t-x)^{\mu-1} = \frac{\Gamma(\nu)\Gamma(\mu)}{\Gamma(\nu+\mu)}t^{\nu+\mu-1}$$

valid for $\mathrm{Re}\,\nu > 0$ and $\mathrm{Re}\,\mu > 0$, it is not difficult to obtain

$$I_2 = \frac{n+2}{6\sigma}\lambda g K_d \Gamma(1-\frac{\epsilon}{\sigma})(2\lambda t)^{-1+\epsilon/\sigma}\,, \tag{A.4}$$

$$I_3 = -\left(\frac{n+2}{3}\right)^2 \lambda (gK_d)^2 \frac{\Gamma^2(1-\frac{\epsilon}{\sigma})}{2\sigma\epsilon} \times \left[\frac{\Gamma^2(1+\frac{\epsilon}{\sigma})}{\Gamma(1+\frac{2\epsilon}{\sigma})} - \frac{1}{2}\right](2\lambda t)^{-1+2\epsilon/\sigma} \tag{A.5}$$

and

$$I_5 = \left(\frac{n+2}{6}\right)^2 \frac{1}{2\sigma\epsilon}\lambda (gK_d)^2 \Gamma^2(1-\frac{\epsilon}{\sigma})(2\lambda t)^{-1+2\epsilon/\sigma}\,. \tag{A.6}$$

By integrating in (A.2) over t' and over the length of p', one finds

$$I_4 = \frac{n+2}{6\sigma}\lambda (gK_d)^2 \Gamma\left(1-\frac{2\epsilon}{\sigma}\right) \times \left(-I_4^{(1)} + I_4^{(2)} + O(\epsilon)\right)(2\lambda t)^{-1+2\epsilon/\sigma}$$

where

$$I_4^{(1)} \equiv K_{\mathrm{d}}^{-1}\int \frac{\mathrm{d}^d x}{(2\pi)^d}\frac{1}{x^\sigma(\mathbf{e}+\mathbf{x})^\sigma};$$

$$I_4^{(2)} \equiv K_d^{-1}\int \frac{\mathrm{d}^d x}{(2\pi)^d}\frac{1}{x^\sigma(\mathbf{e}+\mathbf{x})^\sigma(1+x^\sigma)}\,,$$

with $\mathbf{e}$ the unit vector along the d-axis. The first integral is easily done with the result

$$I_4^{(1)} = \frac{K_d^{-1}}{2^d\pi^{d/2}}\frac{\Gamma^2(\frac{d-\sigma}{2})\Gamma(\frac{2\sigma-d}{2})}{\Gamma^2(\frac{\sigma}{2})\Gamma(d-\sigma)} = \frac{1}{\epsilon} + \frac{1}{2}D_\sigma + O(\epsilon)\,.$$

In order to carry out $I_4^{(2)}$, one can use the following expansion of $1/(\mathbf{e}+\mathbf{x})^\sigma$:

$$(x^2 + 2\mathbf{x}\cdot\mathbf{e} + 1)^{-\sigma/2} = [\max(x,1)]^{-\sigma} \times \sum_{n=0}^{\infty} [\min(x,1/x)]^n (-1)^n c_n^{\sigma/2}(\hat{\mathbf{x}}\cdot\mathbf{e})$$

where $\hat{\mathbf{x}}$ stands for the unit vector of $\mathbf{x}$ and $c_n^{\sigma/2}(\hat{\mathbf{x}}\cdot\mathbf{e})$ are Gegenbauer polynomials. This leads to

$$\int \mathrm{d}\hat{x}(\mathbf{x}+\mathbf{e})^{-\sigma} = \begin{cases} 1 & x \le 1 \\ x^{-\sigma} & x \ge 1 \end{cases}$$

which can be then used to find the leading contribution to $I_4^{(2)}$

$$I_4^{(2)} = \frac{2}{\sigma}\ln 2 + O(\epsilon).$$

Finally, we get

$$I_4 = \frac{n+2}{6\sigma}\lambda (gK_d)^2 \Gamma\left(1-\frac{2\epsilon}{\sigma}\right) \times \left(\frac{2}{\sigma}\ln 2 - \frac{1}{2}D_\sigma - \frac{1}{\epsilon} + O(\epsilon)\right)(2\lambda t)^{\frac{2\epsilon}{\sigma}-1}\,. \tag{A.7}$$

The substitution of equations (A.4–A.7) in (A.1) leads to an explicit expression for $\Gamma_{10}^{(i)}(q=0,t)$ up to terms of order ϵ.

We renormalise now according to (4) the (bare) quantities entering this expression. For the fields s, $\widetilde{s}$ and the coupling constant g a one-loop renormalisation will be sufficient. All the necessary information is available in equations (5–7). The residual singularity is then removed by requiring [1]

$$Z_0^{-1/2}\int_0^\infty \mathrm{d}t \mathrm{e}^{-\mathrm{i}\omega t}\Gamma_{10}^{(i)}(q=0,t)_b = \text{finite for } \epsilon \to 0.$$

Here the subscript b denotes the expression of $\Gamma_{10}^{(\mathrm{i})}$ obtained above in which only bare quantities appear. From this condition we compute Z_0 as given by equation (9).

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